Int. J. Heat Mass Transfer. Vol. 35, No. 10, pp. 2435–2444, 1992
Printed in Great Britain

0017-9310/92 $5.00+0.00
Pergamon Press Ltd

Non-stationary heat transfer and dispersion effects in granular media

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(*Received* 28 *June* 1991)

Abstract—Unsteady heat transfer in a macroscopically isotropic and homogeneous particulate mixture is considered with the help of a general approach based on averaging the local heat conduction equations valid in mixture phases over a configurational ensemble of particles and on ideas of the self-consistent field theory. A closed set of equations for the mean temperatures of the phases is derived by neglecting the direct heat transport through contacts with contiguous particles. Both mean heat flux and interphase exchange are shown to be essentially frequency dependent so that the effective heat conductivity deviates considerably from its stationary value. This is representative of the relaxation processes influencing unsteady heat transfer and generates corresponding dispersion effects. Under the weak non-stationary condition the set can be reduced to either a single 'equivalent' equation belonging to the elliptic type or a system of two simplified equations whose reliability has been discussed in detail previously on the example of heating a motionless granular bed through a flat boundary.

1. INTRODUCTION

THE METHODS available for describing an average unsteady transfer process in heterogeneous media rely on either the usual single-phase parabolic heat conduction equation involving some effective heat conductivity or a phenomenological system of separate transport equations for each phase of a medium with time-independent interphase heat exchange coefficients. That the former approach is entirely inconsistent under non-stationary conditions has been experimentally demonstrated in a convincing way as early as in 1959 [1, 2]. The latter approach amounts to extending a quasi-stationary scheme to a region of heat transfer time scales for which the system as well as its solutions do not obviously hold. A thorough review of recent advances of a two-phase model in problems of practical importance is to be found in refs. [3–5]. Following ref. [3] the phenomenological system is to be written in the form

$$c_0\varepsilon(\partial/\partial t+\mathbf{v}_0\nabla)\tau_0 = -\nabla\mathbf{q}-\sigma,$$

$$c_1\rho(\partial/\partial t+\mathbf{v}_1\nabla)\tau_1 = \sigma,$$

$$\mathbf{q} = -\lambda^*\nabla\tau_0, \quad \sigma = k(\tau_0-\tau_1), \quad k = (\lambda^*/4a^2)A^2, \tag{1}$$

where λ^* is the effective heat conductivity, $\mathbf{q}$ and σ play the roles of mean heat flux and mean heat exchange between the phases per unit volume of a granular medium, respectively. The parameter A is understood to represent a constant independent of physical properties, with $A = 2$ according to ref. [3]. It is important that transient heat flux through the assemblage of contacting particles is neglected so that the second equation in set (1) contains no conduction term.

The quantities λ^* and k are thought of as constants to be determined from experimental data relating to steady or quasi-steady processes. It is just such an assumption which makes set (1) inadequate for the transfer physics because of the full neglect of relaxation of both an instantaneous value of the mean heat flux to a given mean temperature gradient and a current mean temperature of the dispersed phase to that of the continuous matrix. The physical significance of the latter relaxation processes has been explained in refs. [6, 7] whereas implications connected with the former have been pointed out in ref. [8]. These processes result in the discrepancies between the conclusions following from set (1) and experiments. Nevertheless, using set (1) for particular unsteady problems sometimes leads to satisfactory consequences and, anyhow, there is no other alternative but to employ set (1) for lack of a more sound foundation of the non-stationary heat transfer theory for granular media.

Thus, a two-fold task of principal nature arises. It seems to be necessary, firstly, to provide for determining a reasonable basis for theoretical description of the relaxation processes and the resulting dispersion effects accompanying unsteady heat transport in granular and other heterogeneous systems. Secondly, it is desirable to find conditions and to elucidate reasons of approximate validity of set (1) in many essentially unsteady situations.

The study of both these problems constitutes the main purpose of this paper. The first problem is successfully resolved with the help of the technique of averaging over the ensemble of permissible configurations of dispersed particles together with the methods of the theory of self-consistent fields put forward in refs. [9, 10]. This results in a new closed

NOMENCLATURE

A	parameter in equations (1) and (32)
a	radius of particles
c	heat capacity per unit volume
$\mathbf{E}$	mean temperature gradient
F, G	quantities defined in equation (15)
h	interphase heat exchange
k	interphase heat exchange coefficient
M_j, N_j	parameters defined in equation (19)
$\mathbf{q}$	mean heat flux
$\mathbf{r}, \mathbf{R}$	dimensional and dimensionless coordinate vectors
s	parameter introduced in equation (8)
T	relaxation time scale
$\mathbf{v}$	mean velocity.

Greek symbols

α	$= \lambda_1/\lambda_0$
β	$= \lambda/\lambda_0$
ε	$= 1-\rho$
κ	thermal conductivity
λ	heat conductivity
μ	$= \tau_1/\tau$
ρ	dispersed phase concentration by volume
σ	interphase heat exchange
τ	temperature
χ	dimensionless thickness of a layer filled with pure matrix material
ω	Fourier transform parameter.

Subscripts

0	ambient medium (matrix)
1	dispersed phase
s	stationary values.

Superscripts

*	perturbations of mean temperature caused by the test sphere
′	connected with the test sphere
^	field inside the test sphere
0	values of mean temperature and its gradient at the test sphere centre.

system of averaged heat transfer equations attributed to two co-existing continua which model continuous and dispersed phases. The second problem is settled by means of reducing these equations to the form specific for those in set (1) and comparing the coefficients involved.

A mixture consisting of identical spherical particles immersed into an ambient continuous medium is primarily considered as a representative example of granular systems in a broad sense. The main conclusions are shown, however, to remain true for disperse and heterogeneous media of more complicated structure and for mass transfer processes.

2. BASIC EQUATIONS

The averaged equations for the mean temperatures of the continuous and dispersed phases of a macroscopically homogeneous and isotropic medium containing identical spherical particles are to be derived with the help of the ensemble averaging procedure developed in ref. [9]. They happen to be of the same form as the equations in set (1) providing $\mathbf{q}$ and σ are defined in the following manner:

$$\mathbf{q} = -\lambda_0\nabla\tau-(\lambda_1-\lambda_0)\frac{3\rho}{4\pi a^3}\int_{|\mathbf{r}-\mathbf{r}'|\leqslant a}\nabla_{\mathbf{r}}\hat{\tau}(t,\mathbf{r}|\mathbf{r}')\,\mathrm{d}\mathbf{r}'$$

$$\sigma = \frac{3\rho}{4\pi a^3}\int_{|\mathbf{r}-\mathbf{r}'|\leqslant a}\nabla_{\mathbf{r}}\hat{\mathbf{q}}(t,\mathbf{r}|\mathbf{r}')\,\mathrm{d}\mathbf{r}',\quad \tau = \varepsilon\tau_0+\rho\tau_1. \tag{2}$$

Here the number concentration of particles appearing in the corresponding formulae in ref. [9] is expressed through the concentration of the dispersed phase by volume, and the functions $\hat{\tau}(t,\mathbf{r}|\mathbf{r}')$ and $\hat{\mathbf{q}}(t,\mathbf{r}|\mathbf{r}')$ represent the mean temperature and heat flux within a single test sphere. The introduction of these quantities implies averaging over the ensemble of possible arrangements of all the neighbouring particles compatible with the presence of the centre of the test sphere at the point $\mathbf{r}'$.

For reasons given below, it is convenient to introduce new dimensionless coordinates and time by using scales a and a^2/κ_1

$$R = a^{-1}\mathbf{r},\quad Fo = (a^2/\kappa_1)^{-1}t, \tag{3}$$

and, after that, to apply the Fourier transform with respect to the dimensionless time Fo. Then, from equations (1) and (2) the following equations, governing non-stationary heat transfer in the granular medium under study, are received in terms of the Fourier transformations of the unknown variables:

$$c_0\varepsilon\left(\frac{\kappa_1}{a^2}i\omega+\frac{\mathbf{v}_0}{a}\nabla\right)\tau_0 = -\frac{\nabla\mathbf{q}}{a}+h,$$

$$c_1\rho\left(\frac{\kappa_1}{a^2}i\omega+\frac{\mathbf{v}_1}{a}\nabla\right)\tau_1 = -h, \tag{4}$$

where

$$\mathbf{q}(\mathbf{R}) = -\frac{\lambda_0}{a}\nabla\tau-(\lambda_1-\lambda_0)\frac{3\rho}{4\pi a}\int_{X\leqslant 1}\nabla_{\mathbf{R}}\hat{\tau}(t,\mathbf{R}|\mathbf{R}')\,\mathrm{d}\mathbf{R}'$$

$$h(\mathbf{R}) = -\frac{3\rho\lambda_1}{4\pi a^2}\int_{X\leqslant 1}\Delta_{\mathbf{R}}\hat{\tau}(t,\mathbf{R}|\mathbf{R}')\,\mathrm{d}\mathbf{R}',\quad \mathbf{X} = \mathbf{R}-\mathbf{R}' \tag{5}$$

(for simplicity, the same notation is retained for Fourier transformations as for corresponding original quantities).

The closure of the averaged field equations in set (1) is ensured, as is clearly seen from set (2), by expressing the conditioned mean quantities involved in the integrals in set (2) in terms of the unconditioned unknown variables τ_0 and τ_1. A mathematical problem whose solution enables one to do so has been discussed in detail in ref. [10]. If the Peclet number for one particle is small compared with unity, then it is allowable, when employing a coordinate system with an origin at the test particle centre, to neglect terms due to convective transport. Then the problem has to be stated as that of heat conduction both inside and outside the test sphere under the condition of continuity of the temperature and heat flux at its surface. When the conditional ensemble averaging is performed, the test sphere is formally to be regarded as immersed in some fictitious continuum, the properties of which coincide with those of the granular medium far away from the test sphere but depend on the distance from its surface in its vicinity. The character of this dependence is dictated by the manner in which the particles are packed in the medium, that is, by the form of the binary correlation function which determines the relative positions of particles forming a pair.

The solution of such a problem under steady conditions has been undertaken for randomly packed disperse systems in ref. [10] and a good agreement with the available experimental data on the effective stationary conductivity has been reported. Consideration of rather a complicated dependence of the properties of the fictitious medium on coordinates requires a cumbersome numerical calculation to be carried out while solving the test particle problem. What is worse, the calculation may be expected to be valid rigorously only when applied to granular media of certain particular structure which is described by the binary correlation function having been used. In order to simplify the matter and to make the calculation applicable to a broader range of disperse and heterogeneous systems, it is reasonable to have recourse to a suitable semi-empirical model. Such a model may consist of an assumption that the test sphere is surrounded by a homogeneous fictitious medium separated from the sphere surface by a concentric spherical layer filled with pure material of continuous phase. The thickness χa of this layer is unknown beforehand and the coefficient χ plays the role of an empirical parameter to be found afterwards by comparing theoretical results with experimental evidence. In what follows this model is suggested without further comments.

Thus, the ordinary heat conduction equations are valid inside the test sphere and within the concentric layer. Their Fourier transformations are to be written in the form, the independent variables from equations (3) being used,

$$i\omega\hat{\tau} = \Delta\hat{\tau}, \quad i\omega(\kappa_1/\kappa_0)\tau' = \Delta\tau' \tag{6}$$

where $\hat{\tau}$ and τ' are understood to be mean temperatures for $X < 1$ and for $1 < X < 1+\chi$, respectively.

The properties of the fictitious medium when $X > 1+\chi$ are uniform and coincide with those of the granular medium as a whole. It means that heat transport in the region indicated is governed by the equations in set (1). In order to make them closed, one needs to relate $\mathbf{q}$ and σ to the unknown variables. Under steady conditions σ and $\mathbf{q}$ are proportional to $\tau = \tau_0 = \tau_1$ and $\nabla\tau = \nabla\tau_0 = \nabla\tau_1$, respectively, scalar coefficients of proportionality to be calculated with the help of self-consistency equations, that is, by means of comparing the above linear representations for $\mathbf{q}$ and σ with those resulting from definitions in equations (2). In a general case of non-stationary heat transfer the situation is more difficult since $\mathbf{q}$ and σ must be linear functions not only of the mean temperature and its gradient but also of their time derivatives [10]. This difficulty can be avoided, however, by using the Fourier transform when linear relations of the former type are preserved, the coefficients of proportionality being now thought of as dependent on $i\omega$ too. For the quantities in set (4) it will be thus assumed that

$$\mathbf{q} = -(\lambda/a)\nabla\tau, \quad \lambda = \beta\lambda_0, \quad h = -i\omega\rho(\lambda_1/a^2)\mu\tau. \tag{7}$$

By substituting equation (7) into equations (4) and neglecting a convective contribution to the heat flux in the coordinate system connected with the test particle, one gets Fourier transformations of averaged equations governing unsteady heat transfer in the fictitious medium

$$s^2\tau = \Delta\tau, \quad \varepsilon\tau_0 = (1-\rho\mu)\tau, \quad \tau_1 = \mu\tau$$

$$s^2 = \frac{\kappa_1}{\kappa_0}\left[1+\rho\left(\frac{c_1}{c_0}-1\right)\mu\right]\frac{i\omega}{\beta}. \tag{8}$$

Equations of the same type govern non-stationary heat transfer in the original granular medium as well. It follows from them that $|s^2| \sim l^{-2}$, $l = L/a$, L being the linear scale of the mean temperature fields. Evidently, a continual description of heat transfer is adequate only if that scale is much larger than the scale a of the inner structure of the granular medium, that is, if $|s^2| \ll 1$. On the contrary, the linear scale of perturbations induced by the test particle in the fields of mean temperature equals a so that the term on the right-hand side of the first equation in set (8) must be one or a few orders of magnitude larger than the term on the left-hand side. Thus, one is free, in the first approximation, to drop-out the latter term altogether and, consequently, to omit the left-hand side term in the second equation in set (6) as well. This has been done previously in ref. [11] and amounts to using quasi-stationary forms of the transport equations everywhere outside the test particle.

The inequality $|s^2| \ll 1$ imposes a restriction from above on the values of the characteristic frequency ω of a non-stationary heat transfer process which might be investigated on the basis of continual equations. Retaining the term $i\omega\hat{\tau}$ in the heat conduction equation inside the test sphere (the first one in set (6)) means, at the same time, that the inequality

$$(\kappa_1/\kappa_0)[1+\rho(c_1/c_0-1)\mu]\beta^{-1} \ll 1 \qquad (9)$$

has to be true in order that non-stationary dispersion effects could be significant within the scope of a continual manner of the description of unsteady heat transport. For example, it is usually satisfied with good accuracy for granular beds infiltrated with a gas. When inequality (9) is not fulfilled, the unsteady terms of all the heat conduction equations must be taken into account. In this case the role played by relaxation effects outside the test particle is comparable with or even is greater than that of relaxation inside this particle.

In compliance with a general method discussed in refs. [10, 11], one has to introduce perturbations τ^* and τ'^* of the mean temperature field conditioned by the presence of the test sphere as compared with the unconditional one. This gives

$$\tau(\mathbf{R}|\mathbf{R}') = \tau(\mathbf{R})+\tau^*(X|\mathbf{R}'),$$
$$\mathbf{q}(\mathbf{R}|\mathbf{R}') = -\lambda\nabla\tau-\lambda\nabla\tau^*, \quad X > 1+\chi$$
$$\tau'(\mathbf{R}|\mathbf{R}') = \tau(\mathbf{R})+\tau'^*(X|\mathbf{R}'),$$
$$\mathbf{q}'(\mathbf{R}|\mathbf{R}') = -\lambda\nabla\tau-\lambda_0\nabla\tau'^*, \quad 1 < X < 1+\chi. \qquad (10)$$

Then the test particle problem is to be formulated in the form

$$i\omega\hat{\tau} = \Delta\hat{\tau}, \quad 0 < X < 1;$$
$$\Delta\tau'^* = 0, \quad 1 < X < 1+\chi; \quad \Delta\tau^* = 0, \quad X > 1+\chi$$
$$\hat{\tau} = \tau+\tau'^*, \quad \lambda_1\mathbf{n}\nabla\hat{\tau}+\lambda_0\mathbf{n}\nabla\tau'^*, \quad X = 1$$
$$\tau'^* = \tau^*, \quad \lambda_0\mathbf{n}\nabla\tau'^* = \lambda\mathbf{n}\nabla\tau^*, \quad X = 1+\chi$$
$$\tau^* \to 0, \quad X \to \infty; \quad \hat{\tau} < \infty, \quad X = 0 \qquad (11)$$

$\mathbf{n}$ denoting the unit external normal vector on the test sphere surface. The mean temperature $\tau(\mathbf{R})$ can be expressed in the vicinity of the point $\mathbf{R}'$ as a Taylor expansion, that is, as a series in degrees of X_j. A sufficient accuracy can be shown to be provided for in the case under study if only two terms of such a series are retained. Hence

$$\tau(\mathbf{R}) = \tau^0+\mathbf{E}^0\mathbf{X}, \quad \tau^0 = \tau(\mathbf{R}'), \quad \mathbf{E}^0 = \nabla\tau(\mathbf{R}') \qquad (12)$$

both τ^0 and $\mathbf{E}^0$ being regarded as constant quantities.

3. SOLUTION OF THE TEST PARTICLE PROBLEM AND EQUATIONS OF SELF-CONSISTENCY

Solutions of the problem expressed by equations (11) can be sought in the form of expansions in spherical functions. By confining the discussion to the level of accuracy specific to the approximate representation of equations (12), only two terms of any such expansion have to be accounted for. After a simple calculation this yields, in particular

$$\hat{\tau} = [\tau^0 C_0 I_{1/2}(y)P_0+E^0 C_1 I_{3/2}(y)P_1]y^{-1/2},$$
$$y = X(i\omega)^{1/2} \qquad (13)$$

$I_{N/2}(y)$ being the Bessel functions of an imaginary argument and P_0 and P_1 denoting the Legendre polynomials, $P_0 = 1$ and $P_1 = \mathbf{E}^0\mathbf{X}/E^0X$. The coefficients C_j are equal to

$$C_0 = (\pi/2)^{1/2}\,\mathrm{sh}^{-1}(i\omega)^{1/2}$$
$$\times\{1+i\omega\alpha(1+\beta\chi)[3\beta(1+\chi)]^{-1}F\}^{-1}(i\omega)^{1/2}$$
$$C_1 = 3(\pi/2)^{1/2}\,\mathrm{sh}^{-1}(i\omega)^{1/2}[2(\beta^2+1)(G-1)$$
$$+\beta(5G+4)]\{2[G(2\beta+1)-1+\beta]F$$
$$+\alpha(3-2F)[G(2\beta+1)+2-2\beta]\}^{-1} \qquad (14)$$

where a function and parameters are brought forward

$$F = 3(i\omega)^{-1}[(i\omega)^{1/2}\,\mathrm{cth}\,(i\omega)^{1/2}-1],$$
$$\alpha = \lambda_1/\lambda_0, \quad G = (1+\chi)^3.$$

Now the integration in set (5) can be carried out. The result is

$$\int_{X<1}\nabla_{\mathbf{R}}\hat{\tau}(\mathbf{R}|\mathbf{R}')\,\mathrm{d}\mathbf{R}'$$
$$= \tfrac{4}{3}\pi\mathbf{E}^0\left\{(F-1)\left[1+\frac{i\omega\alpha(1+\beta\chi)}{3\beta(1+\chi)}F\right]^{-1}\right.$$
$$+[2(\beta^2+1)(G-1)$$
$$+\beta(5G+4)](2[G(2\beta+1)-1+\beta]$$
$$\left.+\alpha(3/F-2)[G(2\beta+1)+2-2\beta])^{-1}\right\},$$

$$\int_{X<1}\Delta_{\mathbf{R}}\hat{\tau}(\mathbf{R}|\mathbf{R}')\,\mathrm{d}R'$$
$$= \tfrac{4}{3}\pi\tau^0 i\omega F\left[1+\frac{i\omega\alpha(1+\beta\chi)}{3\beta(1+\chi)}F\right]^{-1}. \qquad (15)$$

By substituting equations (15) into the definitions of $\mathbf{q}$ and h in set (5), one obtains formulae which must be compared with those in set (7). This yields the following algebraic equations reflecting the self-consistency requirement

$$\mu = F[1+i\omega\alpha(1+\beta\chi)F/3\beta(1+\chi)]^{-1}$$

$$\beta = 1+(\alpha-1)\rho\left\{(F-1)\left[1+\frac{i\omega\alpha(1+\beta\chi)}{3\beta(1+\chi)}F\right]^{-1}\right.$$

$$+[2(\beta^2+1)(G-1)+\beta(5G+4)](2[G(2\beta+1)-1+\beta]$$

$$\left.+\alpha(3/F-2)[G(2\beta+1)+2-2\beta])^{-1}\right\}. \quad (16)$$

The second equation serves to find β as a function of other parameters whereas the first one determines μ. Both β and μ depend not only on the ratio α of the heat conductivities of the materials of the phases, the volume concentration ρ of the dispersed phase and the structural parameter χ, but also on the dimensionless frequency ω, and just this reveals the physical origin of the dispersion effects influencing non-stationary heat transport. For real unsteady processes, the characteristic values of ω (measured, in accord with equations (3), in units of the dimensional frequency κ_1/a^2) are ordinarily small enough compared with unity. This gives an opportunity to seek a solution of equations (16) in the form of series in degrees of $i\omega$. Below attention is confined only to the first two terms of these series. By means of using

$$\beta = \beta^{(0)} - i\omega\beta^{(1)} + \cdots, \quad \mu = \mu^{(0)} - i\omega\mu^{(1)} + \cdots \quad (17)$$

in equations (16), while expanding these equations into series in degrees of $i\omega$, it can be straightforwardly obtained

$$\beta^{(0)} = 1+\rho(\alpha-1)M_1/N_1,$$

$$\beta^{(1)} = \rho(\alpha-1)(1/15+M_1N_3/N_1^2)$$
$$\times[1+\rho(\alpha-1)(M_1N_2-M_2N_1)/N_1^2]^{-1}$$

$$\mu^{(0)} = 1, \quad \mu^{(1)} = 1/15+\alpha(1+\beta^{(0)}\chi)[3\beta^{(0)}(1+\chi)]^{-1} \quad (18)$$

where the quantities

$$M_1 = (2(\beta^{(0)})^2+2)(G-1)+\beta^{(0)}(5G+4)$$

$$M_2 = 4\beta^{(0)}(G-1)+5G+4$$

$$N_1 = 2(2\beta^{(0)}+1)G-2+2\beta^{(0)}$$
$$+\alpha[(2\beta^{(0)}+1)G+2-2\beta^{(0)}]$$

$$N_2 = 2(1+2G)+2\alpha(G-1)$$

$$N_3 = (\alpha/5)[(2\beta^{(0)}+1)G+2-2\beta^{(0)}] \quad (19)$$

are introduced.

The coefficient $\beta^{(0)}$ determines, in fact, the dimensionless effective heat conductivity of the granular medium in stationary circumstances which have been studied extensively in ref. [10]. It can be used to find a value of the empirical dimensionless thickness χ of the concentric layer around the test sphere by comparing the formula in set (18) with either more strict

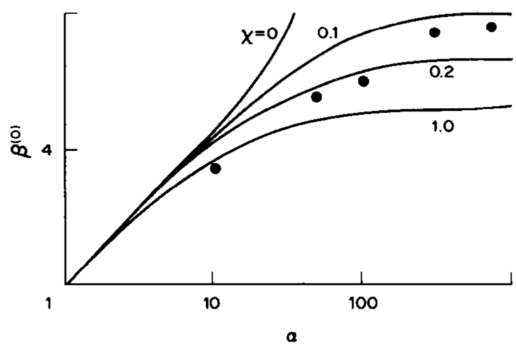

FIG. 1. Effective dimensionless heat conductivity as a function of α at $\rho = 0.6$ and different χ (figures on the curves); points, experiments of ref. [12].

theoretical relations or experimental data. In Fig. 1 curves characterizing the dependence of $\beta^{(0)}$ on α at different χ are presented along with experimental results of ref. [12]. Unfortunately, the comparison does not appear to be sufficiently conclusive, supposedly for the effect of contact conductivity between touching particles having been not excluded in the experiments with polydisperse systems performed in ref. [12]. However, a rough evaluation gives $0.1 \leqslant \chi \leqslant 0.4$. To get a more convincing result applicable to randomly packed monodisperse mixtures with spherical particles, one is able to turn to experiments on the electric conductivity of emulsions in ref. [13] used quite successfully while checking the theory advanced in ref. [10]. Theoretical curves $\beta^{(0)}(\rho)$ are drawn in Fig. 2 for different χ and two values at α. It can be easily seen that the experimental points fall close to the curves corresponding to $\chi \approx 0.3$ at both values of α. Characteristic dependences of $\beta^{(0)}$ upon ρ at $\alpha = 100$ and $\alpha = 0.1$ and various χ are also shown in Fig. 3.

Through carrying out the inverse Fourier transform and returning to the original dimensional independent variables with the help of set (3), the first equations in sets (7) and (17) yield a relation characterizing the relaxation of the heat flux in unsteady processes

$$\mathbf{q} = (1-T_q\partial/\partial t)\mathbf{q}_s,$$
$$\mathbf{q}_s = -\lambda_s\nabla\tau = -\beta^{(0)}\lambda_0\nabla\tau,$$
$$T_q = (\beta^{(1)}/\beta^{(0)})(a^2/\kappa_1). \quad (20)$$

Here $\mathbf{q}_s$ is a quasi-stationary heat flux corresponding to a given instantaneous value of the mean temperature gradient and T_q is the specific relaxation time.

The second and the third equations in set (8) yield with the former accuracy (the equality $\mu^{(0)} = 1$ is allowed for)

$$\tau_1 = [1-i\omega(1-\rho)^{-1}\mu^{(1)}]\tau_0. \quad (21)$$

It reads, in the original variables

$$\tau_1 = (1-T_\tau\partial/\partial t)\tau_0, \quad T_\tau = \mu^{(1)}(1-\rho)^{-1}(a^2/\kappa_1) \quad (22)$$

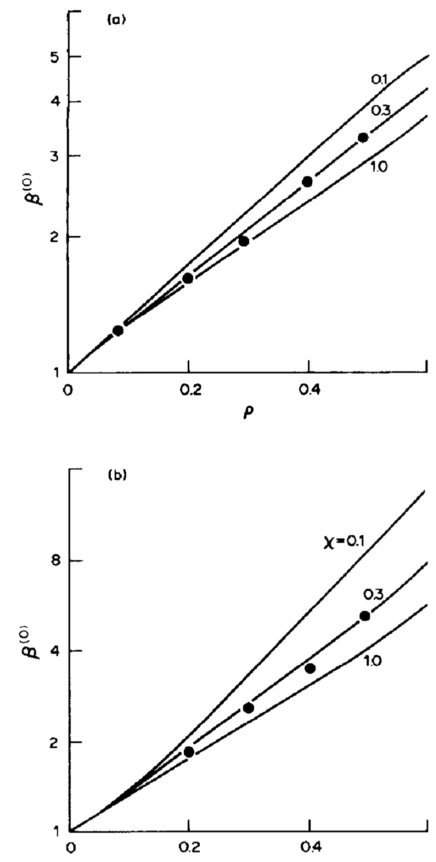

FIG. 2. Theoretical dependence of $\beta^{(0)}$ on ρ at different χ and $\alpha = 15.7$ (a) and 100 (b); points, experiments of ref. [13].

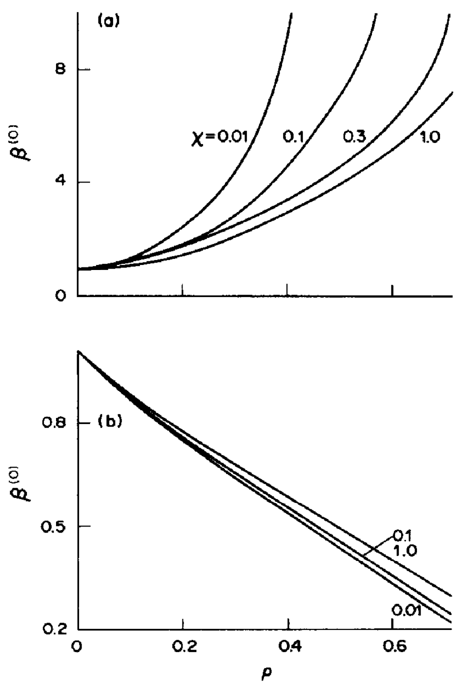

FIG. 3. Dependence of $\beta^{(0)}$ on ρ at different χ (figures on the curves), $\alpha = 100$ (a) and 0.1 (b).

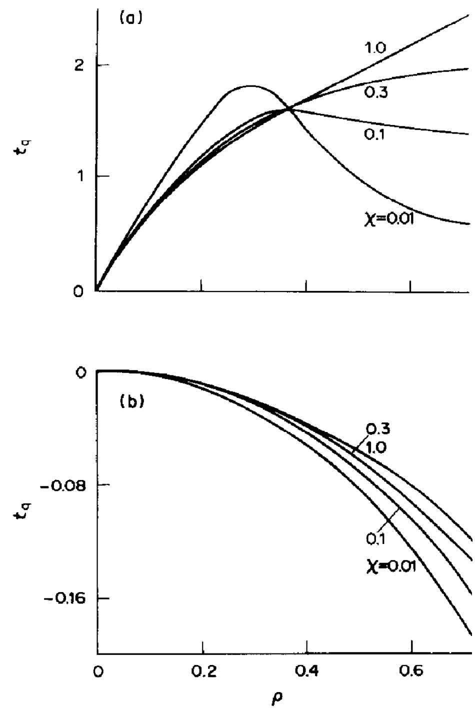

FIG. 4. Dimensionless relaxation time $t_q = \kappa_1 T_q/a^2$ as a function of ρ at different χ, $\alpha = 100$ (a) and 0.1 (b).

T_τ being another relaxation time. This equation describes the relaxation of the mean temperature of the dispersed phase to a given value of that of the continuous one.

A relaxation relation for the mean interphase heat exchange can be obtained easily as well. It follows from equations (8) and (17) after a simple manipulation and allowing for equation (7) that

$$\tau_0 - \tau_1 = i\omega\mu^{(1)}(1-\rho)^{-1}\tau \tag{23}$$

which gives in the original variables

$$h = -\sigma = -k_s(1 - T_h\partial/\partial t)(\tau_0 - \tau_1)$$

$$k_s = -\rho(1-\rho)\lambda_1/\mu^{(1)}a^2, \quad T_h = \mu^{(1)}a^2/\kappa_1. \tag{24}$$

Equations (20), (22) and (24) are rigorous within the limits of their accuracy and, thus, make unnecessary supplementary assumptions concerning the relaxation of relevant quantities in unsteady heat transfer processes. It is worth noting that they are somewhat different from those usually postulated on an empirical basis.

The dependence of dimensionless relaxation times $\kappa_1 T/a^2$ on ρ is illustrated in Figs. 4 and 5 at various χ for $\alpha = 100$ and $\alpha = 0.1$. These figures supplement the information presented in Fig. 3.

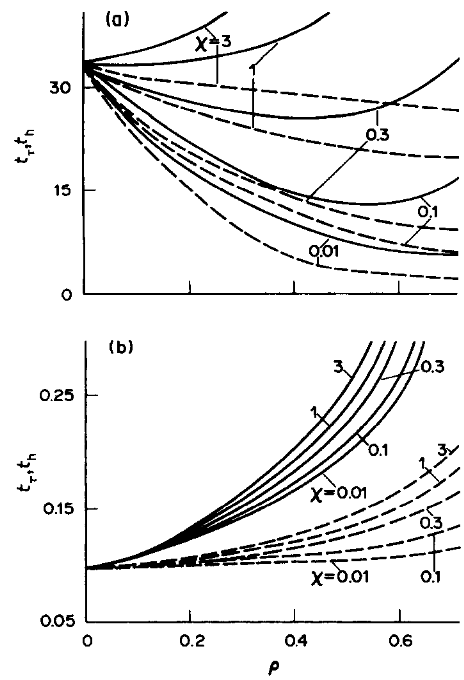

FIG. 5. Dependence of $t_\tau = \kappa_1 T_\tau/a^2$ and $t_h = \kappa_1 T_h/a^2$ on ρ (solid and dashed curves, respectively) at different χ, $\alpha = 100$ (a) and 0.1 (b).

4. CONTINUAL DESCRIPTION OF NON-STATIONARY HEAT TRANSFER

Let us collect together all the conclusions bearing upon the averaged field equations governing unsteady heat transport processes. From equations (1), (20) and (24) a complete set of heat conservation equations and of constitutive relaxation relations is obtained

$$c_0\varepsilon(\partial/\partial t+\mathbf{v}_0\nabla)\tau_0 = -\nabla\mathbf{q}+h$$

$$c_1\rho(\partial/\partial t+\mathbf{v}_1\nabla)\tau = -h, \quad \tau = \varepsilon\tau_0+\rho\tau_1$$

$$\mathbf{q} = -\lambda_s(1-T_q\partial/\partial t)\nabla\tau,$$

$$h = -k_s(1-T_h\partial/\partial t)(\tau_0-\tau_1) \qquad (25)$$

the relaxation times T_q and T_h, the effective stationary heat conductivity $\lambda_s = \beta^{(0)}\lambda_0$ and the quasi-stationary heat exchange coefficient k_s being defined by equations (18), (20) and (24) and in Figs. 3–5. Only set (25) must be used while dealing with non-stationary heat transfer subject to certain conditions imposed above.

It is also expedient to consider simplified methods of the description of heat transport under unsteady conditions. By using an operational expansion, one is able to rewrite the relaxation relation (20) for the mean heat flux with the same accuracy as

$$\mathbf{q}_s = -\lambda_s\nabla\tau = (1-T_q\partial/\partial t)^{-1}\mathbf{q} = (1+T_q\partial/\partial t)\mathbf{q}. \qquad (26)$$

Such a relation has been suggested before as a hypothesis [14]. If both phases of a granular medium are at rest ($\mathbf{v}_0 = \mathbf{v}_1 = 0$) and one does not distinguish between the temperatures τ_0, τ_1 and τ, then summing up the conservation equations in set (25) yields

$$c\partial\tau/\partial t = -\nabla\mathbf{q}, \quad c = \varepsilon c_0+\rho c_1. \qquad (27)$$

Hence, and from equation (26), one gets a hyperbolic equation

$$c(\partial\tau/\partial t+T_q\partial^2\tau/\partial t^2) = \lambda_s\Delta\tau. \qquad (28)$$

This equation is obviously erroneous and cannot be used to characterize unsteady transfer processes since it is incorrect to neglect a similar term of the same order of magnitude in the expression for h while taking into account the relaxation term in the expression for $\mathbf{q}$.

A proper approximate equation for the mean temperature of a motionless granular medium can be derived in the following manner. Firstly, through expanding in degrees of $i\omega$ and taking into account equation (17), one gets from set (8)

$$\beta^{(0)}\Delta\tau = i\omega(\kappa_1/\kappa_0)[1+\rho(c_1/c_0-1)-i\omega H]$$

$$H = \rho\mu^{(1)}(c_1/c_0-1) - (\beta^{(1)}/\beta^{(0)})[1+\rho(c_1/c_0-1)]. \qquad (29)$$

Applying the inverse Fourier transform and passing to variables in equations (3) results in an equation

$$c(\partial\tau/\partial t-T_e\partial^2\tau/\partial t^2) = \lambda_s\Delta\tau, \quad T_e = H(c_1/c)(a^2/\kappa_1) \qquad (30)$$

the thermal capacity c per unit volume of the medium on the whole being expressed in equations (27). The effective relaxation time T_e is illustrated as a function or ρ, χ and α in Fig. 6.

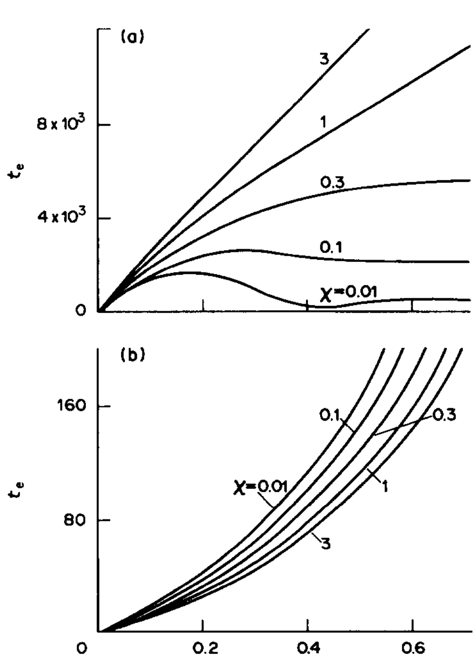

FIG. 6. Effective dimensionless relaxation time $t_e = \kappa_1 T_e/a^2$ as a function of ρ at different χ, $\alpha = 100$ (a) and 0.1 (b).

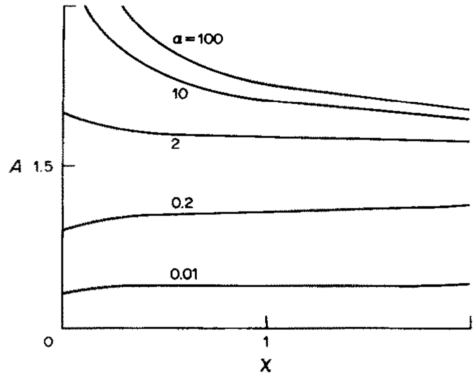

FIG. 7. Dependence of parameter A on α at $\rho = 0.6$ and different χ (figures on the curves).

As T_e is always positive, equation (30), which is to be substituted for equations (28), belongs to the elliptic type. For the first time a single 'equivalent' equation to describe heat extraction from geothermal reservoirs was obtained in ref. [15] for a simple case when heat conduction is completely neglected and the convective heat transport with an infiltrating fluid is only significant. Later on, such equations have been derived and studied in refs. [16, 17].

It is instructive to deduce from the above consideration a set of equations for the phase temperatures, τ_0 and τ_1, similar to equations (1). With this purpose in view, equations (8) will be somewhat rearranged with the help of equations (17) and, as before, only the terms of the zeroth and the first-order in $i\omega$ will be retained. This yields

$$\Delta\tau_0 = (\kappa_1/\kappa_2)[i\omega\varepsilon/\beta^{(0)} + (c_1/c_0)(\rho\varepsilon/\beta^{(0)})(\tau_0-\tau_1)]$$
$$\varepsilon(\tau_0-\tau_1) = i\omega\mu^{(1)}\tau_1. \quad (31)$$

Hence, after using the inverse Fourier transform and returning to the dimensional variables, one obtains for motionless granular media

$$c_0\varepsilon\partial\tau_0/\partial t = \lambda_s\Delta\tau_0 - (\lambda_0/4a^2)A^2(\tau_0-\tau_1)$$
$$c_1\rho\partial\tau_1/\partial t = (\lambda_0/4a^2)A^2(\tau_0-\tau_1)$$
$$A^2 = 12\rho\varepsilon[1/5\alpha + (1+\beta^{(0)}\chi)/\beta^{(0)}(1+\chi)]^{-1}. \quad (32)$$

These equations conform with those in set (1) at $\mathbf{v}_0 = \mathbf{v}_1 = 0$, if λ^* involved in the expressions for $\mathbf{q}$ and for σ is understood as λ_s and λ_0, respectively, and A^2 is defined in accordance with the formula in set (32). The dependence of A upon χ at $\alpha = 100$ and $\alpha = 0.1$ for granular beds ($\rho = 0.6$) is presented in Fig. 7. It can be seen that the value $A = 2$ supposed in ref. [3] represents a certain approximation to actual values of A.

Equations (26)–(32) are derived with reference to media at rest. However, they can be reasonably assigned to a granular medium in motion being considered in the coordinate system connected with its dispersed phase. Then an additional convective term $c_0\varepsilon(\mathbf{v}_0-\mathbf{v}_1)\nabla\tau$ must be included in the left-hand side of equations (30) or a term $c_0\varepsilon(\mathbf{v}_0-\mathbf{v}_1)\nabla\tau_0$ in the first equation in set (32). The transformation to the laboratory coordinate system can be performed easily and, for instance, reduces the equations in set (32) to the form of those in set (1).

Let us conclude this section with an indication that approximate equalities

$$\beta \approx 1, \quad \mu \approx 0 \quad (33)$$

follow from equations (16) within a region of small time scales ($\omega \gg 1$). Although the applicability of the scheme developed above to such a region is questionable, to say the least, equation (33) outlines the essential features of a high-frequency transfer process in the right way. Namely, the particles have not enough time to absorb a considerable amount of heat and do not affect appreciably heat conduction which is run mainly through the continuous phase. Note that set (32) happens to lead to the right asymptotic behaviour at small times if one presumes additionally that $\lambda_s = \lambda_0$.

It must be stressed once more that a prerequisite for continual methods to be applicable for describing non-stationary heat transfer in disperse and heterogeneous media consists in a presumption that a characteristic linear scale of the mean temperature field is much larger than a scale of the inner structure of the medium, $L \gg a$. This leads also, as shown above, to a conclusion that s in equations (8) as well as the left-hand sides of equations are required to be small as compared with unity. The latter condition does not actually hold at the initial stages of many processes, such as heating of a granular bed through a flat boundary, and one has to be aware of the fact while dealing with these stages. Nevertheless, one succeeds sometimes in attaining fair conformity with experiments even during such a stage [3, 18, 19], which seems to be due to occasional reasons and, in particular, to the fact that the approximate equations in set (32) yield correct asymptotics at small times.

5. CONCLUSIONS

The main achievement of this paper consists in a strict derivation of a closed set of both heat conservation and constitutive equations governing nonstationary heat transport within the field of the continual mechanics and physics. The procedure developed gives an opportunity to point out different approximate methods of rough description of unsteady transfer processes which could be of use in various situations. This enables one also to estimate critically the adequacy of phenomenological approaches proposed and used previously and to give an account of the conditions under which these might be successful when applied to particular problems. The most important inference concerns the fact that, whereas the conservation equations are principally of the same basic form as those postulated within such approaches, the constitutive equations happen to be quite different from their versions suggested to describe relaxation processes and various dispersion effects on an empirical basis. This is of particular concern because, as it is evidenced by the above, the form of relaxation relations is able to affect the very type of differential equations called to circumscribe the essential features of unsteady transfer processes.

While dealing with the method developed and, in particular, when trying to extend it to other problems, one should distinguish between the rigorism of the general mathematical technique employed as a foundation and the approximate nature of supplementary assumptions being used within the frames of this technique to simplify the necessary calculations. Two leading assumptions of this kind are to be especially stressed. The first one pertains to the character of relaxation processes progressing in a real granular medium. The thermal inertia of particles has been actually presumed to exceed considerably that of the ambient matrix, and this imposes evident restrictions on the type of media the above analysis is to be applied to. A generalization to a broader range of granular systems does not meet with principal obstacles and amounts to taking into account non-stationary terms of heat conduction equations not only inside the test sphere but also in its exterior. Such a generalization may constitute one of the possible directions of future work.

The second assumption has relevance to the short-range order in the packing of particles. Its point consists in the introduction of a layer filled with pore material of continuous phase so that solving the test particle problem becomes much simpler. If randomly packed granular media are kept in mind, the assumption can be avoided by means of using an appropriate representation for the binary correlation function when formulating the problem in compliance with ref. [10]. This is not so simple, however, for polydisperse systems of particles of an irregular shape and for heterogeneous media of another inner structure. In such cases a satisfactory approximation for the volume concentration of the dispersed phase near any chosen particle is absent and the concept of a layer of pure ambient medium separating their surfaces from a system with uniform properties is, in fact, the only conceivable and quite natural because of not only its simplicity but also for lack of positive knowledge about the features of the short-range order in the packing of the particles. This is the reason why curves corresponding to different χ are drawn along with those for $\chi = 0.3$ in the above figures. One expects large values of χ to be representative of loosely-packed beds of irregular particles whereas low ones play a role when treating heterogeneous systems such as dense-packed polydisperse beds, fractured porous bodies, etc.

As a final remark, we claim all the equations derived to be applicable to non-stationary transport of some other scalar quantities. This statement seems trivial when diffusive transport of an admixture in a granular system is in question. Then one has to put c_0 and c_1 to be identically equal to zero and to substitute λ_0 and λ_1 by diffusivities of the admixture in materials of the phases of the medium. The statement is not so obvious, however, when filtration processes in macroscopically heterogeneous porous media are involved. As a matter of fact, in this case the role of temperature is played by the fluid pressure in fractures and within porous lumps divided by them and pressure conductivity coefficients replace those of heat conductivity. Dispersion and relaxation phenomena are especially significant in this context since time scales of the relaxation be up to several hours or days.

REFERENCES

1. W. K. Harakas and K. O. Beatty, Moving bed heat transfer, *Chem. Engng Progr. Symp. Ser.* **59**, 122–128 (1959).
2. R. Ernst, Der Mechanismus des Wärmeübertragung an Wärmeauschern in Fliesbetten (Wirbelschichten), *Chem.- Ing.- Techn.* **31**, 166–173 (1959).
3. N. V. Antonishin and V. V. Luschchikov, Heat transfer in dispersed media. In *Transport Fluidized Particle Systems*, pp. 71–105. Amsterdam (1989).
4. T. Evangelos and M. Holger, Wärmeübertragung zwischen einer Wand und Dispersion Gas-Feststoff-System, *Chem.- Ing.- Techn.* **61**, 677–686 (1989).
5. Y. C. Chen and D. Luss, Wrong-way behaviour of packed-bed reactors, *A.I.Ch.E. Jl* **35**, 1148–1156 (1989).
6. J. S. M. Botterill, M. H. D. Butt, G. L. Cain and K. A. Redish, The effect of gas and solid properties on the rate of heat transfer to gas-fluidized beds. In *Proc. Int. Symp. Fluidization*, pp. 442–458. Amsterdam (1967).
7. J. S. M. Botterill, *Fluid-Bed Heat Transfer*. Academic Press, London (1975).
8. Yu. A. Buyevich and Yu. A. Korneev, Dispersion of thermal waves in granular material, *Inzh.- fiz. Zh.* **31**, 21–28 (1976) (in Russian).
9. Yu. A. Buyevich, Heat and mass transfer in disperse media—I. Averaged field equations, *Int. J. Heat Mass Transfer* **35**, 2445–2451 (1992).
10. Yu. A. Buyevich, Heat and mass transfer in disperse media—II. Constitutive equations, *Int. J. Heat Mass Transfer* **35**, 2453–2463 (1992).
11. Yu. A. Buyevich and V. A. Ustinov, Non-stationary transfer and dispersion effects in heterogeneous media, *Inzh.-fiz. Zh.* **59**, 807–816 (1990) (in Russian).
12. S. Jagi, D. Kunii and N. Wakao, Heat transfer in packed beds. In *Int. Dev. Heat Transfer*, No. 4, pp. 742–748. ASME, New York (1961).
13. R. E. Meredith and C. W. Tobias, Conductivities in emulsions, *J. Electrochem. Soc.* **108**, 286–290 (1961).
14. N. V. Antonishin, M. A. Geller and A. L. Parnas, A hyperbolic equation of heat conduction, *Inzh.- fiz. Zh.* **26**, 503–508 (1974) (in Russian).
15. N. N. Smirnova, Solution of heat transfer equations for filtration by the method of reducing to a single equivalent equation of heat conduction. In *Physical Hydrodynamics and Heat Exchange*, pp. 61–68. Novosibirsk (1978) (in Russian).
16. Yu. A. Buyevich and E. B. Perminov, Unsteady heating of a motionless granular body, *Inzh.- fiz. Zh.* **37**, 29–37 (1980) (in Russian).
17. V. S. Nustrov and B. N. Saifullaev, Method of equivalent equation in the heat and mass transfer theory, *Inzh.- fiz. Zh.* **54**, 779–786 (1988) (in Russian).

18. A. P. Baskakov, B. V. Berg, A. F. Ryzhkov and N. F. Filippovsky, *Heat and Mass Transfer Processes in a Fluidized Bed*. Izd. Metallurgya, Moscow (1978) (in Russian).

19. V. A. Borodulya, Yu. A. Buyevich and I. I. Markevich, Heat exchange between a solid surface and an infiltrated granular bed. In *Proc. Ninth Int. Heat Transfer Conf.*, Vol. 5, pp. 277–282. Hemisphere, New York (1990).

TRANSFERT THERMIQUE INSTATIONNAIRE ET EFFET DE LA DISPERSION DANS LES MILIEUX POREUX

Résumé—On considère le transfert thermique instationnaire dans un mélange particulaire macroscopiquement isotrope et homogène, à partir d'une approche générale basée sur la moyenne des équations de conduction thermique locale, valables dans les mélanges de phases, pour un ensemble configurationnel de particules, et basé aussi sur la théorie du champ self-consistant. Un système d'équations pour les températures moyennes des phases est obtenu en négligeant le transport direct de chaleur à travers les contacts entre particules contigués. A la fois le flux thermique moyen et l'échange entre phases sont essentiellement dépendants de la fréquence et la conductivité thermique effective s'écarte considérablement de sa valeur stationnaire. Ceci est caractéristique des mécanismes de relaxation qui influencent le transfert thermique instationnaire et génèrent des effets de dispersion. Dans l'instationnarité faible, le système peut être réduit à une seule équation "équivalente" ou à un système de deux équations simplifiées dont la fiabilité est discutée en détail sur l'exemple du chauffage d'un lit granulaire fixe à travers une frontière plane.

INSTATIONÄRE WÄRMEÜBERTRAGUNG UND DISPERSION IN SCHÜTTUNGEN

Zusammenfassung—Die instationäre Wärmeübertragung in einem Gemisch aus makroskopisch isotropen und homogenen Partikeln wird mit Hilfe einer allgemeinen Näherungsmethode untersucht, die auf Gleichungen für die mittlere örtliche Wärmeübertragung in Gemischphasen aus einer Gruppe von Partikeln und auf der Vorstellung der selbstkonsistenten Feldtheorie basiert. Für die mittleren Temperaturen der Phasen ergibt sich ein geschlossener Satz von Gleichungen unter Vernachlässigung der Wärme, die durch den Kontakt benachbarter Partikeln direkt übertragen wird. Sowohl die mittlere Wärmestromdichte als auch der Austausch zwischen den Phasen erweist sich als stark frequenzbhängig, weshalb die effektive Wärmeleitfähigkeit beträchtlich vom stationären Wert abweicht. Dies ist verantwortlich für Relaxationsprozesse, die ihrerseits Einfluß auf die instationäre Wärmeübertragung ausüben und entsprechende Dispersionseffekte verursachen. Bei schwach instationären Bedingungen lassen sich die Gleichungen entweder zu einer einzelnen äquivalenten Gleichung von elliptischem Typ zusammenfassen oder zu einem System aus zwei vereinfachten Gleichungen. Deren Gültigkeit wurde bereits früher anhand eines Beispiels ausführlich diskutiert, bei dem eine ruhende Schüttung durch eine ebene Begrenzung beheizt wird.

НЕСТАЦИОНАРНЫЙ ТЕПЛОПЕРЕНОС И ЭФФЕКТЫ ДИСПЕРСИИ В ГРАНУЛИРОВАННЫХ СРЕДАХ

Аннотация—С помощью общего подхода, основанного на осреднении уравнений локальной теплопроводности, описывающих фазы смеси в ансамбле частиц различной геометрии, и на идеях самосогласованной теории поля, анализируется нестационарный теплоперенос в макроскопически изотропной и однородной смеси макрочастиц. Выведена система замкнутых уравнений для средних температур фаз в пренебрежении прямым теплопереносом через зоны контакта частиц. Показано, что как средний тепловой поток, так и межфазный теплообмен существенно зависят от частоты, что вызывает значительное отклонение эффективной теплопроводности от стационарного значения. Такое поведение характерно для процессов релаксации, влияющих на нестационарный теплоперенос, и приводит к соответствующим дисперсионным эфектам. В условиях слабой нестационарности система уравнений может быть либо сведена к одному "эквивалентному" эллиптическому уравнению, либо к системе двух упрощенных уравнений, надежность которых рассмотрена детально на примере нагрева неподвижного гранулированного слоя через плоскую границу.